Asymmetric market power and wage suppression*

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Abstract:

We study a labor market in which two identical firms compete over a pool of homogenous workers. Firms pre-commit to their outreach to potential employees, either through their informative advertising choices, or through their screening processes, before engaging in a wage competition (à la Bertrand). Although firms are homogeneous, the unique pure-strategy equilibrium is asymmetric: one firm maximizes its outreach whereas the other compromises on a significantly smaller market share. The features of the asymmetric equilibrium extend to a general oligopsony with any finite number of firms.

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1 Introduction

There is now growing empirical evidence alluding to substantial monopsony power possessed by individual firms in the labor market. Manning (2003) and Webber (2015), amongst others, provide evidence based on UK and US data, suggesting that individual firms are faced with upward sloping labor supplies with fairly low elasticity levels, in the range between 0.7 and 1.8, varying by industries. The lack of competition could manifest itself through various channels, including, inter-alia, low rates of union membership, incorporation of non-compete clauses in labor contracts, search frictions, and limited geographic mobility.[1] The evidence provides a possible explanation for the documented persistent decline in the labor share: individual firms exercise their market power to set wages below the marginal product of labor, giving rise to wage markdowns [see Webber (2015), and Cengiz et al. (2019)].

A recent study by Azar et al. (2022) offers a complementary explanation for the documented wage suppression. The study provides compelling and robust estimates for a causal negative equilibrium relationship between market-level concentration, measured by the Herfindahl-Hirschman Index (HHI), calculated based on the share of vacancies and posted real wage rates. In their instrumented (IV) specification, Azar et al. (2022) report a 17% decline in posted wages in response to a shift from the 25^{th} to the 75^{th} percentile in the extent of concentration. They further show that, on average, labor markets are highly concentrated: an average HHI of $3,157$, exceeding the $2,500$-threshold for a high degree of concentration.[2] These findings allude to an important feature that apparently contributes to the documented high degree of labor market concentration: *an asymmetric fragmentation of the market*. The average number of firms in their sample is 20 (see Table 2 therein), whereas the average $3,157$ HHI is roughly equivalent to a symmetric oligopsony of 3 firms. Thus, in many of the local labor markets analyzed, concentration is driven not only by a small number of competing firms, but also by the existence of dominant firms possessing substantially high market shares.

The goal of the current study is to provide a theoretical explanation for the emergence of 'natural' dominant firms in labor markets in line with the asymmetric patterns observed in the data, and further explore the implications for wage suppression. To do so, we augment the classical Bertrand model (where firms simultaneously post wage offers) by introducing a preliminary stage in which firms set their outreach levels. Specifically, we consider a two-stage game of a duopsonistic competition between two identical firms competing over a large pool of homogeneous workers. In the first stage, each firm strategically chooses its outreach to potential employees, either through its informative advertising policy, or through its screening process. In particular, each firm chooses, simultaneously, a fraction of the workers' population to which it will extend a job offer during the second stage. A typical job offer is extended to a single

[1] For further discussion see Krueger (2018) and Azar et al. (2022) amongst others.

[2] Based on the DOJ/FTC horizontal merger guidelines.

worker specifying a single-period wage rate. We allow for wage dispersion at the firm level, so job offers may differ across workers. It is further assumed that job offers are not targeted and, plausibly, are uncoordinated between the firms. A worker that is approached by at least one firm accepts the job offer, or chooses the better one in case both firms provide independent offers. A worker that receives no job offers remains idle.

We show that the *unique* pure-strategy subgame-perfect equilibrium (hereinafter SPNE) of the two-stage game features asymmetric patterns despite the fact that firms are assumed to be ex-ante identical. The market divides into a large firm which maximizes its outreach, and a smaller one which compromises on a significantly lower market share. We further show that, in equilibrium, both firms extend a non-degenerate distribution of wage offers with different expected wages. Specifically, the distribution of wage offers extended by the large firm is stochastically dominated by that extended by its smaller counterpart, and the larger firm offers lower expected wages.

The observed asymmetry between firms seems prima-facie surprising. A firm's natural tendency would be to maximize its outreach so as to enhance its profits. Thus, the choice of the smaller firm to limit its outreach seems counter-productive. However, notice that by choosing to do so, the smaller firm induces its rival firm to alleviate wage competition, knowing that it would still be able to hire workers for substantially lower wage offers. Accordingly, many of the workers would obtain a single offer, effectively rendering the larger firm into a monopsony with respect to these workers. In other words, the ability to pre-commit to a limited market outreach enables a tacit collusion between the firms: one firm gains a monopsony power over a portion of the workers, whereas the other firm is able to recruit workers at lower wages.

Being the only pure-strategy equilibrium configuration of the two-stage game, suggests market dominance (namely, the emergence of a large firm from a pool of ex-ante identical ones) is a natural feature. Indeed, we show that this feature is robust to a natural extension of the model, considering the case of a general oligopsony with a finite number of firms, although uniqueness is not maintained. In the limit when the number of firms diverges, the economy converges to the standard symmetric allocation (as in a classical symmetric Bertrand competition), with full rent-dissipation and no market power possessed by any of the firms.

1.1 Modeling assumptions and related literature

Our study contributes to a voluminous body of literature on imperfect competition in the labor market and monopsony power starting with the pioneering work of Robinson (1933), which has caught some increased attention over the last several years due to growing empirical evidence alluding to substantial market power possessed by firms in the labor market (see our discussion in the introduction). In a perfectly competitive (textbook) labor market each individual firm is faced with a (residual) perfectly elastic supply, meaning it can immediately find a substitute employee for the going market wage rate (and hence has no incentive to raise it above the market benchmark). In turn, the firm would be unable to cut wages even slightly as it would then lose its entire pool of employees who are faced with an infinitely elastic demand for the market wage rate; namely, they can immediately find a substitute job which offers the same level of remuneration. Simply put, monopsony power suggests that an individual firm is faced with an upward sloping (rather than perfectly elastic) supply curve. An extreme manifestation of such market power by firms occurs when a single firm operates in the labor market segment and is effectively faced with the aggregate labor supply.

The literature acknowledges the rarity of such an extreme market scenario but provides both theoretical explanations for the emergence of considerable market power wielded by firms and a rigorous quantitative assessment of the magnitude of such monopsony power. For a small (yet far from complete) sample of the voluminous body of work on imperfect competition in labor markets, see the studies by Burdett and Mortensen (1998), Falch and Strom (2007) and Macedoni (2022), and three review articles by Boal and Ransom (1997), Manning (2011) and Manning (2021) with the references therein. A common explanation for presence of monopsony power is related to the existence of search and matching frictions (see Rogerson et al. (2005) for a comprehensive survey). Such frictions stem from imperfect information about the match idiosyncratic characteristics (not only the level of remuneration) and coordination problems. The fact that a worker needs to invest a considerable amount of time and costly efforts to find a substitute job for his current match creates a rent. The division of this rent between the employer and the worker depends on the bargaining process. However, unless the latter is (implausibly) substantially tilted in favor of the workers, a share (often a significant one) of the surplus is extracted by the employers. This stands in sharp contrast to perfect competition, in which the free entry of firms enables the workers to extract the entire rent.

The search literature considers alternative bargaining protocols, which offer alternative wage determination mechanism. The two canonical frameworks in the literature are the search and matching models of Pissarides (1990) and Mortensen and Pissarides (1994) that assume ex-post asymmetric Nash bargaining following the formation of the match, and the wage-posting model of Burdett and Mortensen (1998) that assume ex-ante take-it-or-leave-it bargaining protocol, in which firms post the wage offers prior to the formation of the match to which they are fully committed, and workers upon a successful match decide whether or not to take the job.

Our model follows the wage posting paradigm but, unlike much of the literature which assumes exogenous matching frictions (typically modeled as Poisson arrival rates of job opportunities), provides a positive explanation for the emergence of endogenous marching frictions. In line with the search literature we obtain wage dispersion (both across and within firms where the latter feature is less common in the literature) despite the fact the both the firms and the workers are ex-ante homogeneous. Moreover, and in line with recent empirical evidence, we demonstrate that the market equilibrium is asymmetric (in terms of the market shares and profitability) although firms are ex-ante identical.

Our modeling choices follow Butters (1977) and Grossman and Shapiro (1984) seminal papers on informative advertising in product markets, in setting focus on the active role of firms in the market (in our context, the formation of jobs). In product markets a consumer typically makes a choice whether or not to purchase the consumption good (say, a brand of orange juice in the local grocery store), without the consent of the seller. In the labor market, in contrast, mutual consent is formative: it is not only a free choice of the worker, who needs to receive a proper job offer from the firm. To emphasize the active role of firms in determining labor-market outcomes, we simplify by assuming that workers are passive and do not engage in search or any other information acquisition activities.[3]

We notice the importance of the uncoordinated nature of outreach choice in our modeling assumptions. Coordination would imply that firms perfectly divide the market into distinct segments, without overlapping, and effectively turn into a local monopsony. Lack of coordination will not, however, bring us to the other extreme case of complete overlapping which yields the standard Bertrand case, in which the entire rent is extracted by the workers (as in the case of perfect competition). This is feasible given our modeling assumptions and would be obtained as a special case of our setup, when outreach is complete (i.e., covering the whole market) for both firms, namely every worker would receive two job offers, one from each firm. As will be shown below, however, this will not form an equilibrium in the two-stage game.

The latter hinges on the presumed ability of the firms to limit its outreach and pre-commit to it. This is a crucial assumption we invoke that is pivotal for our results and hence is worth some further discussion. Notice first, that if firms are unable to commit to their outreach in the first stage, then the equilibrium will collapse to the Bertrand allocation with rents fully extracted by workers. To see this, assume a reversed order in which firms first post their wage offers in the first stage and then set their outreach levels. Notice that after wage offers have been already posted, each firm naturally aims at maximizing its outreach. Anticipating this in advance would imply that both firms will effectively engage in a simultaneous wage competition (a la Bertrand). Second, regarding the order of actions taken by the firm (first target then extend a wage offer) this seems to be in line with actual recruitment processes in the labor market. It is often the case that a vacancy is being posted, possibly stating some fairly broad range of anticipated level of remuneration, using some vague terminology alluding to compensation being commensurate with the applicants' merits and qualifications and negotiable. The specific details of the compensation scheme are typically revealed much later in the screening process. Third, regarding the ability of the firm to commit to its outreach, the firm could do so by fixing its capacity (physical infrastructure). This could take the form of renting limited office space in particular locations by signing on a long term lease contract, or purchasing a limited

[3]The qualitative nature of our results would be maintained if we assume that workers' search costs are sufficiently high.

amount of equipment/ production materials with a substantial lead time of shipment. Finally, limiting the outreach in a modern labor market, where there seems to be a universal (unlimited) online availability to apply for jobs, can take the form of including non-disclosure clauses in labor contracts signed with current employees (thereby limiting matching via professional social networks) and also via screening by the HR officers of the firm, that only invite a fraction of the applicants to follow-up interviews.

Our study bears similarity to the seminal work of Kreps and Scheinkman (1983) who study a duopoly two-stage setup in which the firms pre-commit to their production capacity, and later engage in price competition à la Bertrand. They show that the self-induced capacity constraints allow the firms to avoid the Bertrand paradoxical prediction and derive positive rents.

There are three notable distinctions between the two studies. First, our results indicate that wage dispersion arises on the equilibrium path, whereas their equilibrium supports the Cournot single-price outcome. Naturally, their result is consistent with the law-of-one-price, whereas our prediction is consistent with (commonly observed) wage dispersion in labor markets with information frictions.

Second, although firms are identical and workers are homogeneous ex-ante, our equilibrium outcomes are asymmetric in all aspects – levels of market-outreach, ex-ante distributions of wage offers and as ex-post (realized) wage distributions – all vary across firms. Kreps and Scheinkman (1983), in contrast, prove the existence of a unique symmetric equilibrium where capacity and price levels are identical across firms.

These two differences originate from the crucial third one: market outreach versus capacity constraints. The capacity constraints in Kreps and Scheinkman (1983) are deterministic in the sense that, on the equilibrium path, any amount below the capacity is sold, and no amount exceeding the capacity can be sold. In contrast, the choice of limited market-outreach reduces the probability of hiring. In particular, with a binding capacity constraint, a firm has no incentive to reduce its price, knowing that it can sell its full capacity with probability one, whereas limited outreach levels imply that increasing the wage rate can always increase the likelihood of hiring for the firm. This creates an incentive to increase the wage rate, which essentially induces wage dispersion in equilibrium, despite the fact the firms are ex-ante identical and workers are homogeneous.[4]

1.2 Structure of the paper

The paper is organized as follows. In Section 2 we present the basic set-up. In Section 3 we present the main results: in Subsection 3.1 we characterize the equilibrium in case outreach

[4]The possibility of wage dispersion with homogenous firms and workers is extensively discussed in the search literature and typically relates to the possibility to engage in on-the job search by workers [see Rogerson et al. (2005)]. Notice that we are abstracting from explicitly incorporating search considerations in the model.

levels are exogenous (the second stage of the game), and in Subsection 3.2 we study the case with endogenous outreach levels (the two-stage game). We then provide, in Section 4, three possible extensions: the first relates to multiple firms, the second relates to heterogeneity in productivity among firms, and the last relates to outreach costs. In Section 5 we briefly discuss the effect of asymmetry on the degree of labor market concentration, and its impact on wage suppression. In Section 6 we briefly conclude.

2 The model

Consider a market comprised of two identical firms and a continuum of risk-neutral homogeneous job applicants. Firms employ a linear production function where the productivity of each worker is denoted by $q > 0$. Without loss of generality, we set the workers' outside option and the firms' reservation value to zero. Thus, the formation of a match between a firm and a typical worker is mutually beneficial given any wage level between 0 and q. However, match formation between a potential employee and firm i is limited by an *outreach* level $p_i \in (0, 1]$, such that every firm i approaches only a fraction p_i of the workers. This value represents the informative advertising technology used by the firm [following Butters (1977) and Grossman and Shapiro (1984)]. The case where $p_i < 1$ is referred to as *partial outreach*, compared to *maximal outreach* in case $p_i = 1$.[5]

To determine wages, we assume a standard wage-posting protocol, where each approached worker receives a wage offer from the firm. We allow for wage offers to differ across workers; namely, for wage dispersion at the firm level. Any potential employee can either accept the offer to work for the specified wage offer (with the firm being the residual claimant), or remain idle, in which case the worker collects the reservation wage. In case a potential employee is approached by both firms, he opts for the higher wage offer with a symmetric tie-breaking rule.

More formally, each firm $i \in \{1, 2\}$ dictates a distribution of wage offers given by the CDF $F_i \in \Delta\mathbb{R}_+$. Denote by $w_i \sim F_i$ a random offer from firm i's distribution. Subject to a realized offer $w_i = w$ and given F_{-i}, the expected profit of firm i is

$$\pi_i\left(w|F_{-i}\right) \quad = \quad p_i\left[1 - p_{-i}\left[1 - F_{-i}(w) + \tfrac{1}{2}\Pr(w_{-i} = w)\right]\right](q - w)\,.$$

The term $p_{-i}\left[1 - F_{-i}(w) + \frac{1}{2}\Pr(w_{-i} = w)\right]$ is the probability that a worker is successfully employed by firm $-i$, rather than by firm i, due to an offer of at least w. Assuming that the mass of workers is normalized to unity and with a slight abuse of notation, the expected payoff of firm i is denoted by $\pi_i(F_i|F_{-i}) = \mathbf{E}_{F_i}\left[\pi_i\left(w|F_{-i},\right)\right]$.

We study the aforementioned model as a two-stage game. In the first stage, firms precommit to their outreach to potential employees by setting simultaneously their outreach levels

[5]An alternative interpretation is that the firms follow some preliminary screening process which discards a share of the workforce, prior to the wage-setting stage.

(p_1, p_2), namely the respective fractions of the workers' population to which they extend a wage offer. In the second stage, firms engage in a Bertrand (wage) competition by choosing simultaneously their distributions of wage offers (F_1, F_2). The solution concept we adopt is the Subgame Perfect Nash Equilibrium (SPNE), where a profile $(F_i)_{i=1,2}$ forms an *equilibrium* in the sub-game if $\pi_i(F_i|F_{-i}) \geqslant \pi_i(F_i^*|F_{-i})$, for every distribution F_i^* and firm i.

Two remarks on our modeling assumptions are in order. First, our main focus in this paper is on the positive foundations of monopsony power in the labor market. We abstract from considering product market power. Incorporating the latter would not change the qualitative nature of our results. The possibility to extract rents in product markets would potentially induce firms to limit their output levels and correspondingly to reduce the number of workers they recruit. Second, allowing for the outside options to be bounded away from zero would not change the qualitative features of equilibrium, but would impact the division of the pie between the firms and the workers.

3 Main results

Our analysis is divided into two parts. We first characterize the equilibria in the presence of exogenous outreach levels (Section 3.1), and then extend our model to allow for strategic choices of outreach levels (Section 3.2). A key insight from the first part concerns the generic nature of wage dispersion, and the extent to which wages are suppressed. As it turns out, *any* partial outreach leads to complete wage dispersion at the firm level, and potentially across firms. The main insight from the second part concerns the firms' desire to partially divide the market and not compete over the entire pool of applicants, although the latter is a-priori counter-productive. Moreover, our analysis indicates that there exists a *natural dominant firm*, so that in the unique pure-strategy equilibrium structure, one firm captures a significantly higher portion of the market and supports lower wages (in the sense of stochastic dominance) than the other firm. [6]

3.1 Fixed outreach levels

We start with an analysis of the second stage of the game (i.e., assuming that the outreach levels are fixed), and our first observation concerns the equilibrium under the canonical framework where both firms independently approach *all* potential employees. Evidently, the competition over applicants drives up wages so that workers get their marginal productivity q. In this sense our formulation is completely consistent with the classical prediction. (All proofs are deferred to the Appendix.)

[6] We will revisit the size-wage correlation issue in section 4 below.

Lemma 1. *If $p_1 = p_2 = 1$, then there exists a unique equilibrium where both distributions of wage offers induce only the highest wage level q (i.e., both equal the Dirac measure δ_q).*

The competitive wage level q has specific characteristics that follow from Lemma 1. If firm i can generate a strictly positive payoff, then it will not support wages sufficiently close to the competitive level q, since any atom at q generates a (point-wise) zero payoff. One thus concludes that the competitive wage level q is only supported by one firm if the other firm approaches all employees with a unique wage offer of q.

Next, we present the unique equilibrium structure for any given outreach profile (p_1, p_2). Our analysis suggests that wage dispersion arises both at the firm level (under any partial outreach), and across firms, when outreach levels differ. In this case the distribution of wage offers extended by the firm with the higher outreach level is shown to be stochastically dominated by the distribution of wage offers extended by its lower-outreach rival. Lemma 2 also incorporates, as a special case, the classic set-up characterized in Lemma 1.

Lemma 2. *Given that $0 < p_1 \leqslant p_2 \leqslant 1$, the unique equilibrium is*

$$F_1(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{w(1-p_1)}{p_1(q-w)}, & \text{for } 0 \leqslant w < qp_1, \\ 1, & \text{for } w \geqslant qp_1, \end{cases} \qquad F_2(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{q(p_2-p_1)+w(1-p_2)}{p_2(q-w)}, & \text{for } 0 \leqslant w < qp_1, \\ 1, & \text{for } w \geqslant qp_1, \end{cases}$$

and the expected payoff of firm i is $p_i(1 - \min\{p_1, p_2\})q$.

To fully grasp the economic intuition behind the expected payoff $p_i(1 - \min\{p_1, p_2\})q$, consider the symmetric case and denote $p = p_1 = p_2 < 1$. The value $p(1-p)$ denotes the probability that an applicant is matched only with firm i. As, by presumption, the outside option of an applicant is normalized to zero, firm i can hire the applicant by offering him the minimal wage level, extracting the entire surplus and securing an expected payoff of $p(1 - p)q$. The fact that firms do not approach the entire market limits the extent of competition over the pool of applicants and allows them to derive strictly positive rents.

Notice that under any asymmetric scenario in which, e.g., firm 2 has an advantage over firm 1, reflected in a higher probability of recruiting applicants conditional on both firms making the same wage offer (i.e., $p_2 > p_1$), it sets an atom at the minimal wage level, that is $\Pr(w_2 = 0) > 0$. Moreover, one can easily verify that F_1 (first-order) *stochastically dominates* F_2. So not only firm 2 approaches more applicants, it also offers them lower wages. The reason the larger firm gains from offering lower wages with a higher probability derives from the complete monopsonistic power it possesses with respect to the share of the work-pool that is not targeted by the smaller (rival) firm.

3.2 Strategic outreach levels

We now extend our analysis to allow for strategic choice of outreach levels. Consider the two-stage game described in Section 2, where in the first stage both firms choose simultaneously their desired outreach levels (p_1, p_2), and in the second stage, firms choose their distributions of wage offers (F_1, F_2). The solution concept we adopt is the Subgame Perfect Nash Equilibrium (SPNE). Relying on our previous analysis in Lemma 2, the following theorem characterizes the unique pure-strategy SPNE structure (i.e., up to a switch between p_i and p_{-i}) of the two-stage game.

Theorem 1. *In the unique pure-strategy* SPNE *structure, one firm approaches the entire pool of workers, whereas the other firm approaches only half of them,* $(p_i, p_{-i}) = \left(1, \frac{1}{2}\right)$*, and in the second stage both follow the distributions of wage offers given in* Lemma 2*. Thus, on the equilibrium path, we get*

$$F_i(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{q}{2(q-w)}, & \text{for } 0 \leqslant w < \frac{q}{2}, \\ 1, & \text{for } w \geqslant \frac{q}{2}, \end{cases} \qquad F_{-i}(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{w}{q-w}, & \text{for } 0 \leqslant w < \frac{q}{2}, \\ 1, & \text{for } w \geqslant \frac{q}{2}. \end{cases}$$

Under the given SPNE*, the expected payoffs of firms i and $-i$ are $\frac{q}{2}$ and $\frac{q}{4}$, respectively.*

Two notable insights emerge from Theorem 1. The first concerns the fact that a tacit collusion, in the form of market fragmentation, arises naturally as a unique equilibrium. This division serves as a commitment device to restrain the competition over the pool of workers and ultimately ensure that firms derive positive rents. The pattern of equilibrium where restrained outreach of one firm is reciprocated through reduced wage offers by its rival, is a form of *tacit collusion* between the two firms.

The second, which is somewhat striking, concerns the asymmetric nature of the equilibrium, although firms and workers are homogeneous. The reason for this result is explained as follows. Provided that its rival is committed to substantially limiting its outreach to potential employees, for example below 0.75, a firm's best response would be to maximize its monopsonistic power, given the limited competition in the market, and set its outreach to $p = 1$. This would maximize its recruitment prospects and thereby enhance its expected profits. In contrast, when its rival is committed to the maximal level of outreach, following suit would yield a zero payoff for the firm. Thus, its best response would be to limit its outreach, setting it to $p < 1$, and in our case, the optimum is attained at $p = 0.5$ due to the quadratic functional form of the payoff function.

Barring the pure-strategy asymmetric SPNE given in Theorem 1, there exists also a mixed-strategy symmetric equilibrium, with respect to the outreach levels. Specifically, in the following corollary we provide a mixed-strategy symmetric SPNE with outreach levels fully supported on

$\left[\frac{1}{2}, 1\right]$, and further show that every symmetric mixed-strategy equilibrium must be supported on a dense set in this interval.

Corollary 1. *There exists a symmetric, mixed-strategy* SPNE *so that the outreach levels of both firms are distributed according to*

$$G(p_i) = \begin{cases} 0, & \text{for } p_i < 0, \\ 1 - \frac{1}{4p_i^2}, & \text{for } \frac{1}{2} \leqslant p_i < 1, \\ 1, & \text{for } p_i \geqslant 1, \end{cases}$$

for every $i = 1, 2$, and in the second stage both firms follow the equilibrium profile given in Lemma 2. *Under this* SPNE*, the expected payoffs of both firms are $\frac{q}{4}$. Moreover, in every mixed-strategy symmetric* SPNE*, the outreach levels (p_1, p_2) are supported on a dense set in $\left[\frac{1}{2}, 1\right]$, and the expected outreach is $\frac{3}{4}$.*

The second part of Corollary 1 states that, under any symmetric mixed-strategy SPNE, the lower bound of the outreach levels is 0.5. This implies that in every such equilibrium, the expected payoffs of both firms is $\frac{q}{4}$. Moreover, if the firms' distributions on the outreach levels have density functions, then the stated distribution $G(p_i)$ is the unique symmetric mixed-strategy SPNE.[7] Notice that the current research focuses on the asymmetric pure-strategy equilibrium, rather than on the symmetric mixed-strategy one, due to standard stability concerns regarding the latter.

4 Extensions

In this section we provide three possible extensions to the basic two-firm model. First, in Subsection 4.1, we extend Lemma 2 to markets with multiple firms. We study a setting in which there are $n - 1$ firms with a given outreach level $p \in (0, 1)$, and an additional firm with an outreach level of $p_n > p$. The analysis shows that the sub-game equilibrium described in Lemma 2 extends to the case of multiple firms. Next, In Subsection 4.2, we consider the case of two firms with different productivity levels. We use this setting to study the joint impact of productivity and outreach, both related to the firms' sizes, on the employees' realized wages. Plausibly, allowing for entry of low productivity firms (while maintaining a single firm with a higher productivity level), we illustrate equilibrium configurations in which the more-productive larger firm (in terms of employees and profit) offers higher wages (in expectation), in-line with

[7] Note that we can approximate every distribution with a dense support in $\left[\frac{1}{2}, 1\right]$ by an arbitrary close smooth distribution. This underscores the generic and robust nature of the stated equilibrium.

the empirical findings. Lastly, in Subsection 4.3, we study how outreach costs may impact the equilibrium structure described in Theorem 1.

4.1 The general oligopsony case

In the current subsection we consider an extension of our model to an n-firm setting. Following our basic two-firm set-up, consider the following expected payoff of firm i,

$$\pi_i\left(w|F_{-i}\right) \;=\; p_i\left[\Pi_{j\neq i}\left(1-p_j+p_j\Pr(w_j<w)\right)+\mathcal{T}\left(w,w_{-i},(p_j)_{j=1}^{n}\right)\right](q-w)\,,$$

where $\mathcal{T}(\cdot)$ denotes the probability of successfully employing a potential worker assuming that the same wage offer has been extended by at least one additional firm (as in Section 2, this function is assumed to be symmetric among firms). Under this extended set-up, all firms first decide on outreach levels, and then determine their distributions of wage offers. The following observation extends Lemma 2 and Theorem 1. (The proof is omitted since it is straightforward to verify that the given strategies constitutes an equilibrium.)

Observation 1. *Given $p=p_1=\cdots=p_{n-1}\leqslant p_n\leqslant 1$, there exists an equilibrium where*

$$F_i(w)=\begin{cases}0, & \text{for } w<0,\\ 1-\frac{1}{p}+\frac{1-p}{p}\left(\frac{q}{q-w}\right)^{1/(n-1)}, & \text{for } 0\leqslant w<q\left[1-(1-p)^{n-1}\right],\\ 1, & \text{for } w\geqslant q\left[1-(1-p)^{n-1}\right],\end{cases}$$

for every firm $1\leqslant i\leqslant n-1$, and

$$F_n(w)=\begin{cases}0, & \text{for } w<0,\\ 1-\frac{1}{p_n}+\frac{1-p}{p_n}\left(\frac{q}{q-w}\right)^{1/(n-1)}, & \text{for } 0\leqslant w<q\left[1-(1-p)^{n-1}\right],\\ 1, & \text{for } w\geqslant q\left[1-(1-p)^{n-1}\right],\end{cases}$$

and the expected payoff of firm i is $p_iq\Pi_{j<n}(1-p_j)$. Accordingly, there exists a pure-strategy equilibrium where on the equilibrium path $p_i=\frac{1}{2}$ for every $i<n$, and $p_n=1$, so that the firms follow the above-mentioned distributions.

Observation 1 characterizes one possible equilibrium, extending the equilibrium of the duopsonistic setup to the case with many firms. Two comments are called for. First, the equilibrium in Observation 1 maintains the asymmetric pattern of the duopsonistic case, with one dominant firm gaining a larger market share and offering lower wages (in terms of stochastic dominance) than its rivals. Nevertheless, as the number of firms increases, this asymmetry becomes less pronounced, and in the limit where the number of firms diverges, the asymmetry disappears altogether and the equilibrium converges to the symmetric Bertrand outcome in which firms' rents are fully dissipated. That is, as the number of firms diverges the oligopsonistic equilibrium is converging to the competitive equilibrium.

Second, notice that the formal characterization in Observation 1 does not specify the strategies off the equilibrium path. To support a Nash equilibrium, one could simply assume that off the equilibrium path, all firms deviate to the Bertrand configuration, setting their wage rates at the competitive level, q. Constructing an SPNE, however, is more demanding as it requires that for any profile of outreach levels set in the first stage, the profile of wage distributions set by the firms in the second stage will form an equilibrium in the subgame. There are naturally many such outreach profiles that need to be considered.

To facilitate the exposition of our argument and demonstrate a key difference between the SPNE in the case of two firms and that in the general case involving multiple firms, consider a simplified setup in which the outreach levels in the first stage are chosen from a discrete set rather than from a continuum. Specifically, allow each firm to choose either a full outreach, given by $p = 1$, or a partial outreach, given by $p = 1/2$. Notice that the SPNE for the duopsonistic case is nested in the new setup and is hence unique.

Turning next to the general oligopsonistic case, it is easy to verify that the equilibrium in the second-stage subgame may take one of the two following forms. In the case where the profile of outreach levels set in the first stage involves at most one firm setting its outreach level to unity (full outreach), the wage distributions are specified in the statement of Observation 1. Alternatively, when at least two firms set their outreach levels to unity in the first stage, then the subgame equilibrium in the second stage is given by the Bertrand outcome, where all firms choose to offer the competitive wage rate, q. It then follows that there are two SPNE structures for the two-stage game. One configuration coincides with the equilibrium described in Observation 1, in which one firm chooses a full outreach level, whereas all the other firms set a partial outreach level. In the other configuration, at least three firms are setting their outreach levels at unity. Thus, the uniqueness property of the duopsonistic case does not carry over to the general oligopsonistic case. In particular, there exists a symmetric equilibrium in which each firm sets its outreach at the maximal level and offers the competitive wage rate q. This equilibrium replicates the standard Bertrand result.

4.2 Heterogeneity in productivity and workers' expected wages

In this subsection we extend our basic two-firm model to account for differences in productivity between the firms, and study its impact on the workers' expected wages in each firm. For this purpose, assume that workers in firm $i = 1, 2$ have productivity $q_i > 0$. The following lemma extends Lemma 2 for the case of two firms with asymmetric productivity levels. It shows that the unique equilibrium for the second stage of the game (when outreach levels are given) hinges on the condition $p_i q_{-i} \geqslant p_{-i} q_i$, rather than $p_i \geqslant p_{-i}$, as in Lemma 2 where $q_1 = q_2 = q$. The proof follows directly the arguments in the proof of Lemma 2 and is hence omitted.

Lemma 3. *Fix $p_i \in (0,1)$ and $q_i > 0$ for every firm $i = 1, 2$, and assume (without loss of generality) that $\frac{p_2}{p_1} \geqslant \frac{q_2}{q_1}$. Then, the unique equilibrium is*

$$F_1(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{w(1-p_1)}{p_1(q_2-w)}, & \text{for } 0 \leqslant w < q_2 p_1, \\ 1, & \text{for } w \geqslant q_2 p_1, \end{cases} \qquad F_2(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{w(1-p_2)+p_2 q_1 - p_1 q_2}{p_2(q_1-w)}, & \text{for } 0 \leqslant w < q_2 p_1, \\ 1, & \text{for } w \geqslant q_2 p_1, \end{cases}$$

and the expected payoff of firm i is $p_i(q_i - p_1 q_2)$.

Before turning to analyze the SPNE and the employees' expected wages under this equilibrium, notice how the distributions, and specifically the atom at $w = 0$, vary as a function of $p_i q_{-i}$. In Theorem 1, the higher-outreach firm maintained an atom at $w = 0$, thus supporting a tacit collusion to reduce wages and thereby enabling the limited-outreach firm to hire workers at lower costs. Notably, the existence of an atom at $w = 0$ allows the limited-outreach firm to hire workers for $w = 0$ with a strictly positive probability. In Lemma 3, to differ, this collusion hinges on the combined impact of the firms' productivity and outreach levels. A low productivity gap increases the atom at $w = 0$, as $\Pr(w_2 = 0) = 1 - \frac{p_1 q_2}{p_2 q_1}$, thereby increasing the expected payoff of firm 1, which benefits from the lower wage offers of firm 2.

Let us now focus on the two-stage game in which firms first choose their outreach levels, and then play the sub-game equilibrium described in Lemma 3. Assume, without loss of generality, that $q_2 \geqslant q_1 > 0$. One can easily verify that for every such choice of productivity levels, there exists an equilibrium in which $p_2 = 1$ and $p_1 = q_1/2q_2$.

Observation 2. *For every $q_2 \geqslant q_1 > 0$, there exists a* SPNE *in which $p_2 = 1$ and $p_1 = \frac{q_1}{2q_2}$, and in the second stage both follow the distributions of wage offers given in* Lemma 3.

This SPNE coincides with the one described in Theorem 1 when $q_2 = q_1 = q > 0$. However, this is not necessarily the unique equilibrium. Assuming that $\frac{q_1}{q_2} \in (0.5, 1]$, there exists another equilibrium in which $(p_1, p_2) = (1, q_2/2q_1)$ and the two firms follow the sub-game equilibrium described in Lemma 3, given that $p_2 q_1 < p_1 q_2$. In other words, in case the productivity gap is sufficiently small, there exists an equilibrium in which the low-productivity firm approaches the entire pool of potential employees, whereas the high-productivity firm approaches a smaller portion. Yet, the equilibrium described in Observation 2 is somewhat more robust, because it holds for every choice of $q_2 \geqslant q_1$.

4.2.1 Employees' Expected Wages

The employees' expected wages in both firms depend on the distribution of wage offers (F_1, F_2) and on the outreach levels (p_1, p_2). However, one should not confuse F_i with the wage distribution of firm i's employees, because the selection process dictates that, given two wage offers, employees choose the highest one. So conditional on getting two wage offers, one from each firm, workers in firm i secure an expected wage of $\mathbf{E}[w_i|w_i \geqslant w_{-i}]$ rather than $\mathbf{E}[w_i]$.

Assuming that $q_2 \geqslant q_1$ and considering the more-robust equilibrium $(p_1, p_2) = (q_1/2q_2, 1)$, a straightforward computation shows that the expected wage of employees in firm 1 exceeds that of employees in firm 2. However, this computation hinges on the fact that firm 2 approaches all potential workers, whereas firm 1 approaches less than half. Therefore, firm 2 has monopsony power over most workers, and hence can offer them significantly lower wages, including their reservation wage with probability 0.5. The fact that firm 2 is larger and more productive does not entail offering higher wages in equilibrium, as empirical evidence would suggest [for references on the existence of large firms wage premium, see Brown and Medoff (1989), Idson and Oi (1999) and Colonnelli et al. (2018), amongst others) due to its substantial monopsony power. To reconcile our predictions with the stylized facts, we offer a possible extension which allows for free entry of firms..

The ability of the two firms to suppresses wages via tacit collusion, through limited competition over the pool of potential workers, enables them to maintain positive rents in equilibrium. However, in a general-equilibrium setting, other firms can either enter, or threaten to enter the market, and thereby substantially restrain the ability of the incumbent firms to collude. For example, suppose that a large number of low-productivity firms can enter the market subject to a small entry cost. In order to deter the potential entrants from doing so, the incumbent low productivity firm (firm 1) would need to increase its outreach up to the level which renders the profits virtually zero, namely close to $p_1 = q_1/q_2$. This increase would introduce three significant changes to our previous predictions. First, any increase in p_1 point-wise reduces $F_1(w)$ and $F_2(w)$, in the sense of first-order stochastic dominance. This effect serves to increase the employees' expected wages in both firms, thereby raising their labor share in the aggregate surplus. Second, once p_1 tends towards q_1/q_2, the high-productivity firm would capture a higher portion of the market by concentrating its wage offer distribution close to the lower productivity level q_1. To see this, consider the limit case fixing $p_1 = q_1/q_2$ in the equilibrium wage-offer distributions characterized in Lemma 3 to obtain

$$F_1(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{w(q_2-q_1)}{q_1(q_2-w)}, & \text{for } 0 \leqslant w < q_1, \\ 1, & \text{for } w \geqslant q_1, \end{cases} \qquad F_2(w) = \begin{cases} 0, & \text{for } w < q_1, \\ 1, & \text{for } w \geqslant q_1. \end{cases}$$

Notice that firm 2 supports a unique wage offer of q_1. Third and most importantly, whenever p_1

is sufficiently close to q_1/q_2, the expected wage rate of employees hired by the high-productivity firm 2 would exceed that of the workers hired by the low-productivity firm 1.

To further illustrate the point, consider the following example. Fix $q_2 = 2 > 1 = q_1$ and let the entry cost for a low productivity firm 1 be $\text{EC}_1 = 0.045$. Calculating the wage offer distributions subject to a zero profit condition of the low-productivity potential entrants yields $p_2 = 1$ and $p_1 = 0.45 < 0.5 = \frac{q_1}{q_2}$. Comparing the expected (realized) wage distributions in both firms yields $\mathbf{E}[w_2|w_2 \geqslant w_1] = 0.72 > 0.68 = \mathbf{E}[w_1|w_1 \geqslant w_2]$, where more than 85% of all workers are employed in the high productivity firm 2. Thus, the expected wage rate of hired workers in the high productivity firm strictly exceeds that of hired workers in the low productivity firms, as claimed.

More broadly speaking, any force that shifts the outreach of the low-productivity firm sufficiently close to the point of zero profit, would also ensure that employees secure a higher share of the profits by primarily working for the larger more-productive firm that offers on average higher wage rates.

4.3 Outreach Costs

In the analysis thus far, we have assumed that extending the outreach level is cost-less for the firm. It seems plausible to assume, instead, that extending the outreach entails some costs which could reflect HR activities, referral programs and media publications. The latter is anticipated to limit the optimal outreach levels determined in equilibrium, thereby restraining the competition between the firms over the pool of potential employees with potentially adverse implications for the share of workers in the aggregate surplus. We turn next to examine the implications of positive outreach costs on the equilibrium outcomes.

Consider, for concreteness, a convexly increasing outreach cost of $C(p_i) = p_i^2/2$ for firm i. Following our previous derivations, it is straightforward to show that the chosen outreach levels would shift from $(p_i, p_{-i}) = \left(1, \frac{1}{2}\right)$, as described in Theorem 1, to $(p_i, p_{-i}) = \left(\min\{1, \frac{q(q+1)}{2q+1}\}, \frac{q}{2q+1}\right)$.

Notice that $\min\{1, \frac{q(q+1)}{2q+1}\} > \frac{q}{2q+1}$. Hence, the asymmetric features of the equilibrium are maintained. In general, the outreach levels of both firms decrease (weakly for the large firm when the productivity level, q, is sufficiently high). We turn next to examine the implications of the contraction in the outreach levels of the firms on the labor share in the aggregate surplus.

Assume with no loss in generality that firm i=2 is the one with a higher outreach level in equilibrium. Thus, $1 \geqslant p_2 > p_1$ and $p_1 < 1/2$. By virtue of our previous derivations, the profit level of firm i, i=1,2, is given by $p_i(1-p_1)q$. The aggregate surplus is given by: $[1-(1-p_1)(1-p_2)]q$. Subtracting the sum of the profits derived by both firms from the aggregate surplus yields the workers' surplus, which, following some algebraic manipulations, is given by $p_1^2 q$. The labor share is given by the workers' to aggregate surplus ratio. We will separate between two cases.

Assuming first that the productivity level q satisfies $\frac{q(q+1)}{2q+1} \geqslant 1$, implies that $p_2 = 1$. Hence, the aggregate surplus is given by q as in the case with no outreach costs. The introduction of outreach costs induces, however, a decrease in the outreach of firm 1, so that $p_1 < 1/2$. The latter implies a reduction in the workers' surplus, and hence in the labor share.

Considering next the case where $1 > \frac{q(q+1)}{2q+1}$, it follows that $p_2 = \frac{q(q+1)}{2q+1}$ and $p_1 = \frac{q}{2q+1}$. Notice that in this case the introduction of outreach costs induces a decrease both in the workers' surplus and in the aggregate surplus (the latter due to the limited outreach levels set by both firms). Thus, prima-facie, the effect on the labor share is ambiguous. However, substituting for the outreach levels into the aggregate surplus and the workers' surplus yields, following some algebraic manipulations, that the labor share is given by: $\frac{q}{q^2+4q+2}$. It follows immediately that the labor share is lower than $1/4$ (as $q > 0$), where $1/4$ is the labor share in the absence of outreach costs. We conclude that the introduction of outreach costs gives rise to a reduction in the labor share.

The intuition is straightforward: costly outreach restrains the extent of competition between firms over the pool of potential workers. This in turn is reflected in a corresponding reduction in the labor share.

5 Asymmetric Market Power and Wage Suppression

We conclude by briefly illustrating the effect of asymmetry on the degree of labor market concentration, under the duopsonistic case, and its impact on wage suppression.

To assess the effect of asymmetry we need to define a symmetric benchmark allocation. We comaqre the unique pure-strategy SPNE allocation characterized in Theorem 1 with a symmetric mixed-strategy SPNE allocation in which both firms have an identical market share. In Corollary 1 we provide key characteristics of the mixed-strategy symmetric SPNE. While not proving uniqueness, we show that under any mixed-strategy SPNE, the support of the CDF of outreach levels (for both firms) is given by the interval $\left[\frac{1}{2}, 1\right]$. We further show that the expected outreach of each firm is $\frac{3}{4}$, and its expected payoff is $\frac{q}{4}$.

To conduct a meaningful comparison between the asymmetric and symmetric equilibria, we calibrate the asymmetric pure-strategy SPNE allocation, so that it would induce the same level of employment as in the symmetric mixed-strategy SPNE allocation. To do so we assume that the feasible maximal level of outreach, denoted $p_{\max} \in (0.5, 1)$, is bounded away from 1.[8] Based on the calibration, we compute the degree of concentration, measured by the HHI and based on

[8]As discussed in the previous section, assuming, plausibly, that outreach is costly, is likely to induce firms to limit their outreach levels in equilibrium. The resulting frictional unemployment is consistent with empirical evidence.

the share in producers' surplus, under both equilibrium allocations, and compare the workers' share in the aggregate social surplus.[9]

Denoting by $\hat{p}$ the expected outreach chosen by both firms in the symmetric mixed-strategy regime, the calibration condition is given by:

$$1 - (1 - p_{\max}) \cdot (1 - 0.5) = 1 - (1 - \hat{p})^2,$$

where the expression on the left-hand side is the total employment under the unique pure-strategy SPNE, and the expression on the right-hand side represents the (expected) total employment under the generic symmetric mixed-strategy SPNE. As $\hat{p} = 3/4$, it follows that $p_{\max} = 7/8$.

Starting with the degree of market concentration in the symmetric allocation, we obtain $\text{HHI}^{\text{sym}} = \frac{100^2}{2} = 5,000$. On the other hand, in the asymmetric pure-strategy SPNE, the degree of concentration is given by

$$\text{HHI}^{\text{asym}} = 100^2 \cdot \left[\left(\frac{\frac{p_{\max}}{2}}{\frac{p_{\max}}{2} + \frac{1}{4}} \right)^2 + \left(\frac{\frac{1}{4}}{\frac{p_{\max}}{2} + \frac{1}{4}} \right)^2 \right] = 5,372 > 5,000.$$

Notice that setting $p_{\max} = 0.875$ implies a 6.25% unemployment rate. The resulting concentration index under the asymmetric equilibrium, $\text{HHI}^{\text{asym}} = 5,372$, implies an increase of 7.44% relative to the 5,000 benchmark. As a reference, notice that according to the FTC guidelines, a merger is considered anti-competitive if it increases the HHI measure for concentration by 200 points and brings it above the threshold of significant market concentration, defined as 2,500. Namely, an increase of about 8%.

Next, we calculate the share of workers (WS) in the social surplus under the two regimes. All calculations are straightforward and based on the equilibrium in Lemma 2 and Corollary 1. Starting with the symmetric regime, the share is given by

$$\text{WS}^{\text{sym}} = 0.467.$$

In the asymmetric regime, the share is given by

$$\text{WS}^{\text{asym}} = 0.267.$$

Thus, the asymmetry-driven increase in market concentration results in a 43% reduction in the share of workers in the social surplus!

6 Conclusions

Recent empirical evidence suggests that individual firms wield significant monopsony power in the labor market. These firms use their market power to set wages below the marginal product of

[9]Calculating the HHI, alternatively, based on the outreach levels of the firms, would clearly not change the qualitative nature of our results.

labor, resulting in wage markdowns. This phenomenon potentially accounts for the documented decline in the labor share. Furthermore, the evidence indicates that wage suppression arises not only from a small number of competing firms but also from the asymmetric market structure, characterized by the presence of dominant firms with substantial market shares.

In this study, we propose a possible explanation for the 'natural' emergence of dominant firms in labor markets, consistent with the asymmetric patterns observed in the data. We then use our model to investigate the implications of these patterns for wage suppression.

We begin by introducing a standard symmetric duopsony setup, wherein two identical firms compete for a pool of homogeneous workers by simultaneously posting wage offers. We enhance this setup by adding a preliminary stage where each firm strategically chooses its outreach to potential employees through informative advertising or screening policies.

Our analysis reveals that the unique pure-strategy subgame-perfect equilibrium of this two-stage game exhibits asymmetric patterns, even though the firms are assumed to be ex-ante identical. The market comprises a large firm that maximizes its outreach and a smaller one that settles for a significantly lower market share. We explore the (tacit) collusive mechanism underlying this surprising asymmetric structure, arguing that the smaller firm's decision to accept a lower market share encourages its larger rival to reduce wage competition. We also examine the impact of these asymmetric patterns on wage suppression.

Lastly, we consider three extensions of the basic setup to test the robustness of our qualitative predictions: (i) the general oligopsony case with multiple firms, (ii) an asymmetric duopsony case where firms have different productivity levels, and (iii) the inclusion of outreach costs. We find that the asymmetric patterns persist as the number of firms in the market increases. However, we also demonstrate that, in the limit, as the number of firms approaches infinity, the economy converges to the Bertrand allocation, in which firms offer competitive wage rates and firms' rents are entirely dissipated.

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A Appendices

A.1 Wages are supported on connected sets

Our analysis requires the following supporting lemma which shows that, in equilibrium, both distributions have a convex support and that atoms are only possible at the end points, 0 and q. To be clear, we define an *atom* as a point $w \in [0, q]$ such that $\Pr(w_i = w) > 0$, and F_i is not left-side continuous at w.

Lemma 4. *For fixed values $p_1, p_2 \in (0, 1]$, the wage offers in equilibrium are supported on a connected set with no atoms in $(0, q)$.*

Proof. Fix F_2 and consider the point-wise payoff $\pi_1(w|F_2)$ of firm 1. Assume that F_2 supports an atom $w_0 \in (0, q)$, therefore π_1 is not continuous at w_0 such that $\lim_{w \to w_0^-} \pi_1(w|F_2) < \pi_1(w_0|F_2)$, and firm 1 would not support wage offers below and sufficiently close to w_0. Specifically, for a sufficiently small $\varepsilon > 0$, firm 1 can transfer any positive probability from $(w_0 - \varepsilon, w_0]$ to $w_0 + \varepsilon_0$ where $0 < \varepsilon_0 < \varepsilon$, and strictly increase its payoff due to the discontinuity. But if there exists an $\epsilon > 0$ such that firm 1's strategy does not support wages levels between $w_0 - \varepsilon$ and w_0, then the atom at w_0 is suboptimal. If either $F_1(w_0) > 0$ or $p_1 < 1$, then firm 2 can strictly increase its positive payoff $\pi_2(w_0|F_1) > 0$ by shifting the atom downwards, towards $w_0 - \varepsilon$; this shift reduces costs without affecting the probability to recruit since F_1 is fixed on $(w_0 - \varepsilon, w_0]$. Otherwise, if $F_1(w_0) = 0$ and $p_1 = 1$, then firm 2 cannot hire applicants at wage level w_0 and $\pi_2(w_0|F_1) = 0$. Thus, firm 2 can increase its expected payoff by shifting the atom upwards. The latter deviation provides a strict improvement unless firm 2's expected payoff is necessarily zero at any wage level, which occurs if and only if $F_1(w) = 0$ for every $w < q$. In other words, firm 2 can sustain an interior atom, in equilibrium, if and only if firm 1 follows a Dirac measure at q (a unique wage level of q). However, if firm 1 sustains an atom at q, while firm 2 does not employ a Dirac measure at q, then there exists a wage level $w^* \in [0, q)$ such that $F_2(w^*) > 0$, and $\pi_1(w^*|F_2) > 0 = \pi_1(q|F_2)$,. So, firm 1 has a strictly profitable deviation from q to w^*, and this violates the necessary condition for an interior atom of firm 2. We conclude that no interior atoms exist, and the payoff functions are continuous on $(0, q]$, where continuity at q follows from the $(q - w)$ term of π_i.

We now prove that the distributions are supported on a connected set. Assume there exists an open interval $I = (w_-, w_+) \subset [0, q]$ such that $\Pr(w_2 \in I) = 0$, while $0 < F_2(w_-) < 1$. By the elimination of interior atoms, we can take the maximal I that sustains the above conditions. That is, we take the maximal interval I such that for any other interval $I_0 \subset [0, q]$ where $I \subsetneq I_0$, it follows that $\Pr(w_2 \in I_0) > 0$. Since F_2 is constant on I while w increases, it follows that π_1 is linearly decreasing on $[w_-, w_+]$ and $\pi_1(w_-|F_2) > \pi_1(w_+|F_2)$. Note that w_+ is generally not an atom of F_2 unless $w_+ = q$, which ensures a linear decrease towards zero, in any case. So for some small $\varepsilon > 0$, firm 1 would not support wage levels in $[w_+, w_+ + \varepsilon)$, as these wage levels are strictly dominated by wage levels in I, sufficiently close to w_-. However, the maximal choice of I suggests that the interval $[w_+, w_+ + \varepsilon)$ is supported by firm 2 with positive probability. Thus, firm 2 has a strictly positive deviation of shifting these wage levels downwards. Therefore, we conclude that such I does not exist, and both wage distributions are supported on a connected set, as needed. ∎

A.2 The canonical result: proof of Lemma 1

Lemma 1. *If $p_1 = p_2 = 1$, then there exists a unique equilibrium where both distributions of wage offers induce only the highest wage level q (i.e., both equal the Dirac measure δ_q).*

Proof. Fix $p_1 = p_2 = 1$. The point-wise payoff of firm 1, given F_2, is $\pi_1(w|F_2) = \left[F_2(w) - \frac{\Pr(w_2=w)}{2}\right](q - w)$. If F_2 supports a unique wage level of q, then firm 1's weakly dominant strategy is to follow the same Dirac measure, establishing an equilibrium where both get a zero expected payoff. Any other strategy of firm 1 would provide a profitable deviation to firm 2, so there exists no other equilibrium where $\Pr(w_i = q) = 1$. Moreover, the indifference principle suggests that, in equilibrium, an atom at q exists only if the maximal expected payoff is zero, thus no other equilibrium exists such that $\Pr(w_i = q) > 0$.

We move on to prove uniqueness under the assumption that $\Pr(w_i = q) = 0$ for both firms. First, we eliminate the possibility of having an atom at 0. Assume that $\Pr(w_2 = 0) > 0$. If $\Pr(w_1 = 0) > 0$, then either firm can shift the atom upwards and profit by the increased probability of recruiting. Moreover, if only one firm supports an atom at 0, there is a zero probability to recruit applicants at this level, and the point-wise payoff is zero. Again, the indifference principle suggests that the maximal expected payoff at any wage level would also be zero, which leads to a unique atom at q, and the above-mentioned equilibrium.

Thus far we have established that any alternative equilibrium has no atoms, so the continuous payoff functions are given by $\pi_i(w|F_{-i}) = F_{-i}(w)(q - w)$. One can easily verify that F_1 and F_2 have the same support, similarly to the the proof of Lemma 4. Denote the support by I_0, and assume there exists a wage level $w \in I_0$ such that $0 < F_2(w) < 1$. This implies that the point-wise payoff at w and the expected payoff $\mathbf{E}[\pi_1(w_1|F_2)]$ of firm 1 are strictly positive. However, the fact $F_2(\inf I_0) = 0$ suggests that $\pi_1(\inf I_0|F_2) = 0$. By continuity, one can take a small $\varepsilon > 0$ such that $\pi_1(w|F_2) < \mathbf{E}[\pi_1(w_1|F_2)]$ for every $w \in I_1 = [\inf I_0, \inf I_0 + \varepsilon)$. This implies that the wage levels in I_1 are suboptimal for firm 1, but $\Pr(w_1 \in I_1) > 0$. A contradiction. We conclude that no alternative equilibrium exists, as stated. ∎

A.3 Proof of Theorem 1

Lemma 2. *Given that* $0 < p_1 \leqslant p_2 \leqslant 1$*, the unique equilibrium is*

$$F_1(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{w(1-p_1)}{p_1(q-w)}, & \text{for } 0 \leqslant w < qp_1, \\ 1, & \text{for } w \geqslant qp_1, \end{cases} \qquad F_2(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{q(p_2-p_1)+w(1-p_2)}{p_2(q-w)}, & \text{for } 0 \leqslant w < qp_1, \\ 1, & \text{for } w \geqslant qp_1, \end{cases}$$

and the expected payoff of firm i *is* $p_i(1 - \min\{p_1, p_2\})q$*.*

Proof. We first compute the point-wise and expected payoffs of both firms to establish an equilibrium, and later prove uniqueness. Note that the given strategies are well-defined as CDFs, both supported on $[0, qp_1]$, where F_1 is non-atomic and F_2 potentially has an atom of size $1 - \frac{p_1}{p_2}$ at $w = 0$. Given (F_1, F_2), the point-wise payoff functions are

$$\begin{aligned} \pi_1(w|F_2) &= p_1\left[1 - p_2\left[1 - F_2(w) + \tfrac{1}{2}\Pr(w_2 = w)\right]\right](q-w), \\ \pi_2(w|F_1) &= p_2\left[1 - p_1\left[1 - F_1(w)\right]\right](q-w). \end{aligned}$$

For $w \in (0, qp_1]$, the point-wise payoff of firm 1 is

$$\begin{aligned} \pi_1(w|F_2) &= p_1\left[1 - p_2\left[1 - F_2(w)\right]\right](q-w) \\ &= p_1\left[1 - p_2\left[1 - \frac{q(p_2-p_1)+w(1-p_2)}{p_2(q-w)}\right]\right](q-w) \\ &= p_1\left[(1-p_2)(q-w) + q(p_2-p_1) + w(1-p_2)\right] \\ &= p_1(1-p_1)\,q, \end{aligned}$$

and the payoff is independent of w, establishing the indifference principle for any positive-measure set of valuations in $[0, qp_1]$. A similar computation for $w = 0$ would show $\pi_1(0|F_2) < qp_1(1-p_1)$. The latter inequality does not contradict the equilibrium statement since $\Pr(w_1 = 0) = 0$ and zero-measure suboptimal outcomes do no affect the expected payoff. Also, any wage offer above qp_1 is suboptimal, since it leads to higher wage levels without increasing the probability of recruiting an employee (by the fact that $F_i(pq_1) = 1$).

Similarly, for every $w \in [0, qp_1]$, the point-wise payoff of firm 2 is

$$\begin{aligned} \pi_2(w|F_1) &= p_2\left[1 - p_1\left[1 - F_1(w)\right]\right](q-w) \\ &= p_2\left[1 - p_1\left[1 - \frac{w(1-p_1)}{p_1(q-w)}\right]\right](q-w) \\ &= p_2\left[(1-p_1)(q-w) + w(1-p_1)\right] \\ &= p_2(1-p_1)\,q. \end{aligned}$$

Again, the payoff is independent of w, and similar arguments (as noted for firm 1) hold for firm 2.

We move on to prove uniqueness. In case $p_1 = 1$, we revert back to Lemma 1. The statement of Lemma 1 is embedded in the current one, so we can assume that $p_1 < 1$. Assume, to the contrary, that a different equilibrium (F_1, F_2) exists. We know from Lemma 4 that the distributions have no atoms at $(0, q)$ and the supports are connected sets.

We first focus on the least upper bound of the supports. Firm 2 can secure an expected payoff of at least $p_2(1-p_1)q$ by fixing a Dirac measure at $w = 0$ (denote this measure δ_0). Therefore it will not support an atom at $w = q$, which produces a point-wise zero payoff. Using left-side continuity and the fact the support is connected, firm 2 will not support any wage levels close to q, thus firm 1 cannot support these wage levels as well. That is, wage levels close to q produce a point-wise payoff close to 0, while a strictly positive payoff for both firms can be secured by taking wage levels bounded away from q. We conclude that both firms have a strictly positive expected payoff, in equilibrium, while the least upper bound is strictly below q.

Let us now show that both distributions are supported on the same set of valuations.[10] Denote the support of F_i by I_i such that $\inf I_i = \underline{w_i}$ and $\sup I_i = \overline{w_i}$. If either $\underline{w_1} \neq \underline{w_2}$ or $\overline{w_2} \neq \overline{w_1}$, then one firm has a strictly decreasing payoff function at the high or low wage levels (the probability to recruit applicants remains fixed while wages increase). By Lemma 4 we know that both distributions are supported on a connected (positive-measure) set of valuations, so the latter conjecture yields a suboptimal expected payoff. We deduce that both distributions have the same support.

Denote $\underline{w} = \inf I_i$ and $\overline{w} = \sup I_i$, and let us prove that $\underline{w} = 0$. Assume that $\underline{w} > 0$. In that case, $\underline{w}$ is not an atom (by Lemma 4) and $F_i(\underline{w}) = 0$. Using left-side continuity, we get $\lim_{w \to \underline{w}^+} \pi_2(w|F_1) = p_2(1-p_1)(q-\underline{w})$, which is strictly less than $p_2(1-p_1)q$ that firm 2 can secure with δ_0. Hence, both distributions are necessarily supported on $\underline{w} = 0 < \overline{w} < q$. In addition, note that the profile of strategies where both firms support an atom at 0 cannot be an equilibrium, since each firm would revert to an infinitesimal increase, due to the discontinuity of the payoff function. So, we need to analyse the remaining possibilities of either no atoms, or a single atom for only one firm.

Consider the case where firm 1 does not have an atom at 0. We can employ the indifference principle for firm 2 over connected positive-measure sets, subject to F_1. The payoff function of firm 2 is continuous and point-wise equals $\pi_2(0|F_1) = p_2(1-p_1)q$. The fact there are no atoms above $w = 0$ implies left-side continuity of the payoff function. Along with the indifference principle, it follows that the same point-wise payoff must hold throughout the support of F_2, specifically for $w \to \overline{w}^+$. Therefore,

$$\pi_2(\overline{w}|F_1) = p_2\left[1 - p_1\left[1 - F_1(\overline{w}) + \tfrac{1}{2}\Pr(w_1 = \overline{w})\right]\right](q-\overline{w}) = p_2(q-\overline{w}) = p_2(1-p_1)q,$$

[10] We remind the reader that all statements hold almost surely, with probability 1.

and $\overline{w} = qp_1$. Similarly, for every $0 \leqslant w \leqslant qp_1$, we get

$$\begin{aligned} p_2(1-p_1)q &= \pi_2(w|F_1) \\ &= p_2\left[1 - p_1\left[1 - F_1(w) + \tfrac{1}{2}\Pr(w_1 = w)\right]\right](q-w) \\ &= p_2\left[1 - p_1 + p_1 F_1(w)\right](q-w)\,, \end{aligned}$$

which leads to $F_1(w) = \frac{w(1-p_1)}{p_1(q-w)}$, as already stated.

Now take $F_2(w)$ and the maximal wage level $\overline{w} = qp_1$. Left-side continuity and the indifference principle yield

$$\pi_1(\overline{w}|F_2) = p_1\left[1 - p_2\left[1 - F_2(\overline{w}) + \tfrac{1}{2}\Pr(w_2 = \overline{w})\right]\right](q - \overline{w}) = p_1(q - qp_1) = p_1(1-p_1)q.$$

Applying the same reasoning as before, the point-wise payoff $p_1(1-p_1)q$ must hold throughout the support of F_1. The latter statement holds up to a zero-measure set w.r.t. F_1 (which does not have an atom at $w = 0$ by assumption), so there is no problem with the evident discontinuity at $w = 0$, generated by the symmetric tie-breaking rule. Specifically, for every $0 < w \leqslant pq$, we get

$$\begin{aligned} p_1(1-p_1)q &= \pi_1(w|F_2) \\ &= p_1\left[1 - p_2\left[1 - F_1(w) + \tfrac{1}{2}\Pr(w_2 = w)\right]\right](q-w) \\ &= p_1\left[1 - p_2 + p_2 F_2(w)\right](q-w)\,, \end{aligned}$$

which leads to $F_2(w) = \frac{q(p_2-p_1)+w(1-p_2)}{p_2(q-w)}$. Note that $\Pr(w_2 = 0) = 1 - \frac{p_1}{p_2} \geqslant 0$, so $p_1 < p_2$ leads to an atom of F_2 at $w = 0$.

We should now consider the other possibility where firm 1 supports an atom at $w = 0$. Denote $a = \Pr(w_1 = 0) > 0$. Since both firms cannot simultaneously have an atom at $w = 0$, we can use the continuity of π_1 and the indifference principle on connected positive-measure sets to compare $\pi_1(0|F_2)$ and $\pi_1(\overline{w}|F_2)$. Namely,

$$\begin{aligned} p_1(q-\overline{w}) &= \pi_1(\overline{w}|F_2) \\ &= \pi_1(0|F_2) \\ &= p_1\left[1 - p_2\left[1 - F_2(0) + \tfrac{1}{2}\Pr(w_2 = 0)\right]\right](q-0) \\ &= p_1\left[1 - p_2\left[1 - 0\right]\right]q, \end{aligned}$$

which yields $\overline{w} = qp_2$. A similar comparison of $\lim_{w\to 0^+} \pi_2(w|F_1)$ and $\pi_2(\overline{w}|F_1)$, which follows from right-side continuity at $w = 0$, and left-side continuity at $w = \overline{w}$, yields

$$\begin{aligned} p_2(q-\overline{w}) &= \pi_2(\overline{w}|F_1) \\ &= \lim_{w\to 0^+} \pi_2(w|F_1) \\ &= \lim_{w\to 0^+} p_2\left[1 - p_1\left[1 - F_1(w) + \tfrac{1}{2}\Pr(w_1 = w)\right]\right](q-w) \\ &= p_2\left[1 - p_1\left[1 - F_1(0)\right]\right](q-0) \\ &= p_2\left[1 - p_1\left[1 - a\right]\right]q. \end{aligned}$$

Thus, $\overline{w} = qp_1(1-a)$. Since both distributions have the same support, we get $qp_1(1-a) = \overline{w} = qp_2$, and $p_2 = p_1(1-a) < p_1$. A contradiction to the initial condition of $p_1 \leqslant p_2$. In conclusion, F_1 is non-atomic whenever $p_1 \leqslant p_2$, and uniqueness follows. ■

Theorem 1. *In the unique pure-strategy* SPNE *structure, one firm approaches the entire pool of workers, whereas the other firm approaches only half of them,* $(p_i, p_{-i}) = \left(1, \frac{1}{2}\right)$, *and in the second stage both follow the distributions of wage offers given in* Lemma 2*. Thus, on the equilibrium path, we get*

$$F_i(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{q}{2(q-w)}, & \text{for } 0 \leqslant w < \frac{q}{2}, \\ 1, & \text{for } w \geqslant \frac{q}{2}, \end{cases} \qquad F_{-i}(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{w}{q-w}, & \text{for } 0 \leqslant w < \frac{q}{2}, \\ 1, & \text{for } w \geqslant \frac{q}{2}. \end{cases}$$

Under the given SPNE*, the expected payoffs of firms i and $-i$ are $\frac{q}{2}$ and $\frac{q}{4}$, respectively.*

Proof. For every outreach profile (p_1, p_2), Lemma 2 states that firm i's unique equilibrium expected payoff is $p_i(1 - \min\{p_1, p_2\})q$. By this uniqueness outcome and the use of a SPNE, we can restrict the analysis to the preliminary stage of choosing the outreach levels. Hence, we consider an auxiliary one-stage game where firms simultaneously choose (p_1, p_2) and firm i's payoff is $p_i(1 - \min\{p_1, p_2\})$. Given $p_{-i} \leqslant 1$, the best response of firm i is either to play $p_i = 1 \geqslant p_{-i}$, which generates a payoff of $1 - p_{-i}$, or to choose some value $p_i < p_{-i}$, which yields a payoff of $p_i(1 - p_i)$. So, for $p_2 = 1$ the best response of firm 1 is $p_1 = 0.5$, and symmetry suggests that the best response of firm 2 is $p_2 = 1$, which establishes an equilibrium. Now fix a profile $(p_1, p_2) \neq (0.5, 1)$. Clearly $p_1 = p_2 = 1$ is not an equilibrium so we can ignore this possibility. Assume, w.l.o.g., that $p_1 \leqslant p_2$. Again, the best response of firm 2 is $p_2 = 1$, and then firm 1 would deviate to $p_1 = 0.5$. We revert back to the only possibility where one firm chooses an outreach of half and the other chooses a maximal outreach, thus concluding the proof. ■

A.4 Proof of Corollary 1

Corollary 1. *There exists a symmetric, mixed-strategy* SPNE *so that the outreach levels of both firms are distributed according to*

$$G(p_i) = \begin{cases} 0, & \text{for } p_i < 0, \\ 1 - \frac{1}{4p_i^2}, & \text{for } \frac{1}{2} \leqslant p_i < 1, \\ 1, & \text{for } p_i \geqslant 1, \end{cases}$$

for every $i = 1, 2$*, and in the second stage both firms follow the equilibrium profile given in* Lemma 2. *Under this* SPNE*, the expected payoffs of both firms are* $\frac{q}{4}$*. Moreover, in every mixed-strategy symmetric* SPNE*, the outreach levels* (p_1, p_2) *are supported on a dense set in* $\left[\frac{1}{2}, 1\right]$*, and the expected outreach is* $\frac{3}{4}$*.*

Proof. Let us first show that the given distribution supports a symmetric equilibrium. To be clear, the probabilities and expectations are taken with respect to $p_{-i} \sim G$, given than p_i is fixed. The expected payoffs given outreach levels of $p_i = 0.5$ and $p_i = 1$ are

$$\begin{aligned} \mathbf{E}[0.5(1 - \min\{0.5, p_{-i}\})q] &= \mathbf{E}[0.5(1 - 0.5)q] = \frac{q}{4}, \\ \mathbf{E}[1(1 - \min\{1, p_{-i}\})q] &= \mathbf{E}[(1 - p_{-i})q] = \frac{q}{4}, \end{aligned}$$

given that $\mathbf{E}[p_{-i}] = \frac{3}{4}$. For every $p_i \in (0.5, 1)$, we get

$$\begin{aligned} \mathbf{E}[p_i(1 - \min\{p_i, p_{-i}\})q] &= p_i q - p_i^2 q \Pr(p_{-i} \geqslant p_i) - p_i q \int_{0.5}^{p_i} \frac{k}{2k^3} dk = \\ &= p_i q - p_i^2 q \left(1 - \left(1 - \frac{1}{4p_i^2}\right)\right) - p_i q \int_{0.5}^{p_i} \frac{1}{2k^2} dk \\ &= p_i q - p_i^2 q \cdot \frac{1}{4p_i^2} - p_i q \left[-\frac{1}{2p_i} + 1\right] \\ &= p_i q - \frac{q}{4} + \frac{q}{2} - p_i = \frac{q}{4}, \end{aligned}$$

as needed.

Now consider a mixed-strategy equilibrium and assume that the lower bound $\underline{p}$ of the support of p_i is strictly above 0.5. Due to the payoff structure of $\mathbf{E}[\underline{p}(1 - \min\{\underline{p}, p_{-i}\})q]$, we get $\underline{p}(1 - \underline{p})q < \frac{q}{4}$. Thus, under continuity and the indifference principle, there exists a strictly profitable deviation to 0.5. Similarly, given an upper bound $\overline{p} < 1$, we get an expected payoff of $\mathbf{E}[\overline{p}(1 - \min\{\overline{p}, p_{-i}\})q] = \overline{p}q(1 - \mathbf{E}[p_{-i}])$, and again there exists a strictly profitable deviation to $\overline{p} = 1$. Following the indifference principle, this also proves that $\mathbf{E}[p_{-i}] = 3/4$ in every mixed-strategy equilibrium, to sustain the same expected payoff given $\underline{p} = 0.5$. Note that every outreach level below 0.5 is also strictly dominated by $p_i = 0.5$, by similar arguments.

We move on to prove that the support of p_i is dense in $\left[\frac{1}{2}, 1\right]$. Assume that there exists an open interval $(a, b) \subsetneq [0.5, 1]$ so that $\Pr(p_i \in (a, b)) = 0$. Take $0 \ll \epsilon < b - a$, and compute that expected payoffs given $p_i = a, a + \varepsilon, b - \varepsilon, b$, so that w.l.o.g. the expected payoffs in a and b are at least as high as the ones in $a + \varepsilon$ and $b - \varepsilon$. By subtracting the terms, we get

$$\begin{aligned} 0 &\leqslant \mathbf{E}[b(1 - \min\{b, p_{-i}\})q] - \mathbf{E}[(b - \varepsilon)(1 - \min\{b - \varepsilon, p_{-i}\})q] \\ &= \varepsilon q + q(b - \varepsilon - b)\mathbf{E}[p_{-i}\mathbf{1}_{\{p_{-i} < b\}}] + q[(b - \varepsilon)^2 - b^2]\Pr(p_{-i} \geqslant b) \\ &= \varepsilon q \left[1 - \mathbf{E}[p_{-i}\mathbf{1}_{\{p_{-i} < b\}}] + [-2b + \varepsilon]\Pr(p_{-i} \geqslant b)\right] \\ &= \varepsilon q \left[1 - \mathbf{E}[p_{-i}\mathbf{1}_{\{p_{-i} \leqslant a\}}] + [-2b + \varepsilon]\Pr(p_{-i} \geqslant b)\right], \end{aligned}$$

where the last equality follows from the fact that $\Pr(p_i \in (a,b)) = 0$, so

$$1 - \mathbf{E}[p_{-i}\mathbf{1}_{\{p_{-i} \leqslant a\}}] + [-2b + \varepsilon]\Pr(p_{-i} \geqslant b) \geqslant 0,$$

and

$$\begin{aligned} 0 &\leqslant \mathbf{E}[a(1 - \min\{a, p_{-i}\})q] - \mathbf{E}_{p_{-i} \sim G}[(a+\varepsilon)(1 - \min\{a+\varepsilon, p_{-i}\})q] \\ &= -\varepsilon q + q(a + \varepsilon - a)\mathbf{E}[p_{-i}\mathbf{1}_{\{p_{-i} \leqslant a\}}] + q[(a+\varepsilon)^2 - b^2]\Pr(p_{-i} > a) \\ &= \varepsilon q \left[-1 + \mathbf{E}[p_{-i}\mathbf{1}_{\{p_{-i} < b\}}] + [2a + \varepsilon]\Pr(p_{-i} \geqslant b)\right] \\ &= \varepsilon q \left[-1 + \mathbf{E}[p_{-i}\mathbf{1}_{\{p_{-i} \leqslant a\}}] + [2a + \varepsilon]\Pr(p_{-i} \geqslant b)\right], \end{aligned}$$

which yields

$$1 - \mathbf{E}[p_{-i}\mathbf{1}_{\{p_{-i} \leqslant a\}}] - [2a + \varepsilon]\Pr(p_{-i} \geqslant b) \leqslant 0.$$

Comparing both inequalities, we conclude that

$$\begin{aligned} 1 - \mathbf{E}[p_{-i}\mathbf{1}_{\{p_{-i} \leqslant a\}}] - [2a + \varepsilon]\Pr(p_{-i} \geqslant b) &\leqslant 1 - \mathbf{E}[p_{-i}\mathbf{1}_{\{p_{-i} \leqslant a\}}] + [-2b + \varepsilon]\Pr(p_{-i} \geqslant b) \\ -[2a + \varepsilon]\Pr(p_{-i} \geqslant b) &\leqslant [-2b + \varepsilon]\Pr(p_{-i} \geqslant b) \\ b - a &\leqslant \varepsilon, \end{aligned}$$

contradicting fact that $\epsilon < b - a$. Thus, the support of p_i is dense in $[0.5, 1]$, as stated. ■